

Information Design with Agency*

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Abstract

We consider a general information design problem in which the task of running a procedure generating information for a continuation game is performed by an agent. A moral hazard problem therefore emerges in which the principal faces a trade-off between generating information that is persuasive in the continuation game, and efficiently incentivizing the agent to comply with the procedure designed. Standard concavification techniques do not apply in this environment. We provide a general methodology to tackle such problems, and examine the way in which moral hazard affects the optimal procedure of the principal.

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1 Introduction

Consider a principal designing an information production procedure (an “experiment”) with the aim of influencing subsequent decisions by a group of receivers. For example, a university may choose a testing and grading policy to influence the job placement of its students, or the management of a firm may choose an information acquisition procedure to improve its decision making. In each case, the actual task of generating information is delegated to an agent: teachers grade students on behalf of schools and universities; similarly, experts, consultants or employees acquire information in order to help managerial decision making. Whenever information acquisition procedures are not contractible, the agent must be incentivized in order to follow the procedure designed by the principal. For instance, the agent might be familiar with the procedure he has used in the past, in which case learning a new procedure causes disutility. We seek to understand how this affects the principal’s design problem.

We propose a tractable model of the kind of principal-agent-receiver interaction described above. The principal designs a procedure generating messages about an unknown state of the world, as well as monetary transfers to the agent. The agent chooses between adopting the procedure designed by the principal and sticking with a given, default, procedure. Switching to the new procedure designed by the principal implies a cost for the agent. We assume that the agent’s choice of procedure is not contractible. However, procedures generate direct or indirect outputs that are contractible. The principal could for example condition transfers on the message that is generated by the procedure used, or on the decisions receivers take as a consequence. The point of departure of our analysis is that the contractibles are the messages, however we show that under certain conditions this approach enables us to capture other forms of contractibility as well.

The substantial assumption of our setup is that the procedure designed by the principal must use as messages the “natural” language used by the default procedure. In practice, information is often conveyed through a natural language: grading, for example, must be on a scale from A to F , consultants must provide specific action recommendations, etc. Without this assumption, the moral hazard problem would disappear, as ensuring that messages differ across procedures would effectively restore their contractibility. By contrast, when procedures share a common language the agent’s choice of procedure cannot be perfectly inferred from the message that it generates. This constraint creates a trade-off for the principal between designing a procedure that generates information about the state of the world so as to influence receivers, and making the designed procedure easy to distinguish from the default one so as

to reduce the cost of agency.

As this agency cost is not linear in the distribution of posterior beliefs induced by the procedure, the problem of the principal cannot be solved by concavification. However, we show how a solution may be obtained by reformulating the problem of the principal as finding a binary splitting of the prior belief that maximizes a simple objective function made up of (a) an “informational payoff” in the continuation game and (b) the agency cost of the procedure designed by the principal. The greater the agent’s switching cost, the more weight the principal attaches to (b) and, therefore, the more informational payoff the principal optimally sacrifices in order to reduce the agency cost. For sufficiently high switching cost, the optimal procedure is completely uninformative.

The paper is organized as follows. Section 2 presents the baseline model, and Section 3 the analysis. An example illustrating our results is presented in Section 4. The proof of the main theorem is in Section 5. Section 6 concludes.

Related Literature. Our paper is in the information design tradition of [Kamenica and Gentzkow \(2011\)](#) and belongs to a recent research program where the design of information not only affects decisions *downstream* of message production, but also shapes incentives for a third party to make non-contractible choices *upstream* of message production ([Rosar, 2017](#); [Bloedel and Segal, 2018](#); [Lipnowski, Mathevet and Wei, 2018](#); [Perez-Richet and Skreta, 2018](#); [Bizzotto, Rüdiger and Vigier, 2019](#); [Zapechelnyuk, 2019](#)). A different kind of moral hazard problem is considered in [Boleslavsky and Kim \(2019\)](#), where the distribution of the state is affected by the unobservable effort of an agent. In our setting, the distribution of the state is fixed, but an agent must be incentivized to run a given procedure. These features connect our work both to [Yoder \(2019\)](#) and to [Rappoport and Somma \(2017\)](#). Our environments differ in that we do not require the principal to be able to contract on posterior beliefs formed by the receivers; on the other hand our model is more restrictive in terms of deviations available to the agent. More broadly, our paper is connected to the literature exploring how to motivate information acquisition that includes [Szalay \(2005\)](#), [Zermeño \(2011\)](#), [Chade and Kovrijnykh \(2016\)](#), and [Angelucci \(2017\)](#), among many others.

2 Model

We consider an information design environment in which the final information of a continuation game is determined by a principal-agent interaction (she and he, respectively). The finite set

of states of the world is denoted Ω , with typical element ω . The common prior μ_0 has full support.¹ An information production procedure (henceforth *procedure* for short) run by the agent provides public information about the realized state to a group of receivers; this group may comprise the principal, but does not include the agent. The agent privately chooses between two procedures, φ and ψ . The procedure φ is exogenous, whereas ψ is designed by the principal; we shall refer to φ as the *default* procedure and to ψ as the *designed* procedure. Based on information generated by the procedure run by the agent, the receivers form a posterior belief $\mu \in \Delta\Omega$ and play a principal-preferred equilibrium action profile of the continuation game that induces a payoff $v(\mu)$ for the principal. This payoff function summarizes all we need to know about the continuation game.²

All feasible procedures generate messages from the finite set M with typical element m , according to probability mass functions conditional on the realized state. We assume $|M| \geq |\Omega| + 1$. To shorten notation, the (unconditional) probability that the default procedure φ generates the message m will be denoted $\phi(m)$. We also define $\underline{\phi} := \min_m \phi(m)$ and assume, up to a redefinition of M , that $\underline{\phi} > 0$. We can think of M as the natural language for the problem at stake. To pursue this idea further, we allow for the possibility that messages also have a natural meaning, captured by a compact and convex set of beliefs $\mathcal{M}(m)$ that each message m may convey, with $\Delta\Omega = \bigcup_{m \in M} \mathcal{M}(m)$. We can then define additional language constraints on the principal by requiring ψ to be such that, for any m generated with positive probability under ψ , the posterior belief $\mu(m; \psi)$ induced by m belongs to $\mathcal{M}(m)$. A natural interpretation is that language constraints prevent the principal from designing procedures that alter the customary meaning of messages. For example, whenever messages are recommendations of equilibrium play in the continuation game language constraints could require that procedures match messages (i.e. action recommendations) to beliefs at which the recommended action profile is an equilibrium of the continuation game.³

The agent can run the default procedure at zero cost. Running the designed procedure on the other hand induces disutility $c > 0$. In line with the interpretation given in the introduction, we refer to c as the agent's switching cost. The agent's choice of procedure is

¹The analysis can be extended to the case of heterogeneous priors with full support using the transformation in [Alonso and Câmara \(2016\)](#) or [Laclau and Renou \(2016\)](#).

²The assumption that receivers play a principal-preferred equilibrium implies that $v(\cdot)$ is upper semicontinuous.

³Messages might have other meanings as well. For example, the set of available messages could be rooted in the states of the world, so that $\Omega \subset M$. A possible language constraint could then be that the message ω be matched to beliefs at which ω is the most likely state.

not contractible, giving rise to moral hazard. To solve this problem, the principal can provide the agent with incentives through a message-contingent transfer scheme $t : M \rightarrow \mathbb{R}_+$.⁴

The timing is as follows. First, the principal chooses ψ and t . Second, the agent decides whether to use ψ or φ . Third, the state of the world is realized, and a message is generated according to the procedure used by the agent. Fourth, receivers play the continuation game (after having observed the contract offered by the principal, and the message generated by the procedure run by the agent). The principal and the agent are risk-neutral, and the equilibrium concept is subgame perfect equilibrium.

2.1 Discussion of Modelling Assumptions

The model we propose builds on the assumption that the principal contracts on the messages that procedures generate. However, incorporating language constraints enables us to capture other forms of contractibility.

Contracting on actions. The principal might be able to directly contract on the actions of the receivers, so that the agent ends up being paid based on what players actually do in the continuation game. In our framework, denoting by A the set of action profiles in the continuation game, this amounts to choosing $M = A$, and letting $\mathcal{M}(a)$ represent the set of beliefs at which a is an equilibrium profile. Note that the contractible actions framework requires additional assumptions if the principal is one of the receivers, as the receiver-principal might otherwise be tempted to choose an action that is not optimal given her posterior belief so as to reduce her payment to the agent.

Contracting on beliefs. In other applications some posterior beliefs might be indistinguishable from one another for contracting purposes, thereby inducing a covering $\{\Delta_k\}_{k=1,\dots,K}$ of the belief space $\Delta\Omega$, where each Δ_k is a contractible region of the belief space. In our framework, this amounts to choosing $M = \{m_k\}_{k=1,\dots,K}$ and $\mathcal{M}(m_k) = \Delta_k, \forall k$. [Rappoport and Somma \(2017\)](#) and [Yoder \(2019\)](#), for instance, allow posterior beliefs to be contracted upon.

⁴Limited liability is key to our main trade-off. Without it, it is possible to show that, for any procedure ψ inducing a message distribution different than ϕ , a transfer scheme t exists ensuring that the incentive constraint holds, the agent's expected payoff is 0, and the principal's expected payment is c .

3 Analysis

This section begins by analyzing the baseline setting in which $\mathcal{M}(m) = \Delta\Omega$ for every $m \in M$, that is, in which the only constraint imposed on the principal is such that ψ must use the same set of messages as φ . We examine in the second part of this section how adding language constraints modifies the analysis.

3.1 Baseline Setting

In the baseline setting, the problem of the principal is to solve for the optimal procedure and transfer scheme among those that are incentive compatible for the agent. This problem yields⁵

$$V(\mu_0) := \max_{\psi, t} \sum_{\omega, m} \mu_0(\omega) \psi(m|\omega) \{v(\mu(m; \psi)) - t(m)\} \quad (\text{P0})$$

$$\text{s.t.} \quad \sum_{\omega, m} \mu_0(\omega) \psi(m|\omega) t(m) - c \geq \sum_m \phi(m) t(m). \quad (\text{IC0})$$

If $V(\mu_0)$ is greater than the principal's expected payoff under the default procedure, then the principal designs a procedure ψ giving her expected payoff $V(\mu_0)$; otherwise, the principal sticks with the default procedure.

We say that a belief distribution $\tau \in \Delta\Delta\Omega$ is a *splitting* of μ_0 (Aumann, Maschler and Stearns, 1995) if it satisfies the Bayes plausibility condition $\sum_{\mu} \tau(\mu) \mu = \mu_0$. The problem (P0) of the principal is conveniently reformulated in terms of the choice of a splitting τ of μ_0 .⁶ Let $T(\mu)$ denote the set of splittings of μ supported on $|M|$ beliefs at most, and $T_v(\mu)$ the set of v -concavifying splittings of μ supported on no more than $|\Omega|$ beliefs:

$$T_v(\mu) := \left\{ \tau \in T(\mu) : |\text{supp}(\tau)| \leq |\Omega|, \sum_{\mu'} \tau(\mu') v(\mu') = \hat{v}(\mu) \right\},$$

where \hat{v} denotes the concavification of v .⁷ One shows that for any μ , $T_v(\mu) \neq \emptyset$. Lastly, let $\bar{\tau} := \max_{\mu \in \text{supp}(\tau)} \tau(\mu)$ denote the probability of the most likely belief under τ .

We now show that a solution of the program (P0) can be obtained by way of the following Split-Match-Pay (henceforth SMP) construction:

⁵Throughout the paper a transfer scheme t is understood as satisfying limited liability of the agent.

⁶In the absence of agency (if $c = 0$, for example) this problem reduces to the classic information design problem of Kamenica and Gentzkow (2011).

⁷The concavification of $v(\cdot)$ is the smallest concave function $\hat{v}(\cdot)$ such that $\hat{v}(\mu) \geq v(\mu)$ for all $\mu \in \Delta\Omega$.

1. **Split:** Choose a binary splitting of μ_0 between a *payment belief* μ^\dagger , generated with probability $p > \underline{\phi}$, and a *resplitting belief* $\hat{\mu}$. Then resplit $\hat{\mu}$ according to $\alpha \in T_v(\hat{\mu})$.
2. **Match:** Construct a corresponding procedure by matching the payment belief μ^\dagger with a message that is least likely under φ , and match other beliefs to messages indifferently.
3. **Pay:** Pay the agent exclusively for generating the message matched to the payment belief μ^\dagger , in such a way that the agent is indifferent between using ψ and φ .

The broad intuition behind the SMP construction is as follows. The agent is risk neutral, so paying him at a single message realization at which the likelihood that he used ψ is maximal minimizes the cost of inducing the agent to use the procedure designed by the principal. By the same logic, the principal may reduce said cost by matching the most likely belief induced by ψ to the least likely message of φ . But then, if the agent is paid at a single belief, conditional on *not* reaching this belief, the principal is now free to generate information in any possible way. It ensues that the problem of the principal can be reduced to the choice of a binary splitting of μ_0 into beliefs μ^\dagger and $\hat{\mu}$ solving

$$\max_{p > \underline{\phi}, \mu^\dagger, \hat{\mu}} \quad pv(\mu^\dagger) + (1-p)\hat{v}(\hat{\mu}) - [c + \gamma(p)] \quad (\text{P})$$

$$\text{s.t.} \quad p\mu^\dagger + (1-p)\hat{\mu} = \mu_0, \quad (\text{BP})$$

where $\gamma(p) := \frac{c\phi}{p-\underline{\phi}}$. We refer to $\gamma(\cdot)$ as the *agency cost* function. This cost function is unlike any cost function encountered in the literature on information design with costs.⁸ In particular, in our setting the agency cost of a procedure is not linear in the splitting of μ_0 associated with this procedure. Therefore, the principal's problem cannot be formulated as a concavification problem.

To any binary splitting $(p, \mu^\dagger, \hat{\mu})$ of μ_0 and any $\alpha \in T_v(\hat{\mu})$ such that $\mu^\dagger \notin \text{supp}(\alpha)$, we can associate a splitting $\tau \in T(\mu_0)$ given by $\tau(\mu^\dagger) = p$ and $\tau(\mu) = (1-p)\alpha(\mu)$ for all $\mu \in \text{supp}(\alpha)$. Then, given an arbitrary collection $\{m_\mu\}_{\mu \in \text{supp}(\tau)}$ of distinct messages from M such that $\phi(m_{\mu^\dagger}) = \underline{\phi}$, we say that the pair (ψ, t) is SMP-associated with $(p, \mu^\dagger, \hat{\mu})$ if, for all $\mu \in \text{supp}(\tau)$,

$$\psi(m_\mu|\omega) = \tau(\mu) \frac{\mu(\omega)}{\mu_0(\omega)}, \quad (1)$$

⁸See e.g. [Gentzkow and Kamenica \(2014\)](#).

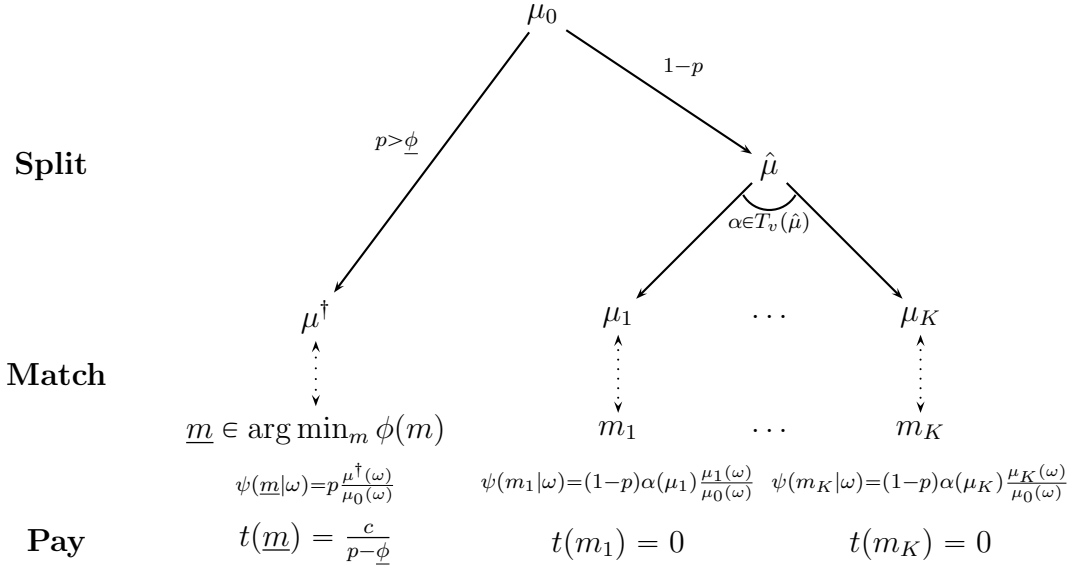


FIGURE 1: THE SPLIT-MATCH-PAY CONSTRUCTION

and

$$t(m_\mu) = \begin{cases} \frac{c}{p-\underline{\phi}} & \text{if } \mu = \mu^\dagger \\ 0 & \text{otherwise.} \end{cases}$$

The SMP construction is illustrated in Figure 1. We can now state our main result.

Theorem 1. *There exist a binary splitting $(p, \mu^\dagger, \hat{\mu})$ of μ_0 solving (P), and a pair (ψ, t) that is SMP-associated with this splitting. Any such (ψ, t) solves (P0).*

We provide in Theorem 1 a general methodology for solving the principal's design problem. A proof of the theorem is provided in Section 5. Our goal in the rest of this section is to examine the way in which moral hazard affects the optimal procedure of the principal. We aim to show, more specifically, that the probability p with which the principal optimally rewards the agent (Theorem 1) can be viewed as balancing the principal's gain from controlling information in the continuation game against her loss from making ψ harder to distinguish from φ .

As a first step, for any $p \in [0, 1]$ define the value

$$I(p) := \max_{\tau \in T(\mu_0)} \sum_{\mu} \tau(\mu) v(\mu) \\ \text{s.t. } \bar{\tau} \geq p$$

representing the highest expected continuation payoff that the principal can obtain whenever some posterior belief must be generated with probability at least equal to p . Clearly, $I(1) = v(\mu_0)$, and since lowering p loosens the constraint in the program highlighted above, $I(p)$ is a non-increasing function of p . Moreover, said constraint is mute whenever $p \leq \max_{\tau \in T_v(\mu_0)} \bar{\tau}$. Hence, $I(p) = \hat{v}(\mu_0)$ whenever this condition holds.

We can now make precise the sense in which, at the optimum, p balances the principal's gain from generating persuasive information for the continuation game against her loss from making ψ harder to distinguish from φ .

Proposition 1. *The probability p with which the principal optimally rewards the agent maximizes $I(p') - \gamma(p')$ over $p' \in (\underline{\phi}, 1]$.*

This characterization leads to easy comparative statics, listed in the next proposition. Intuitively, raising the agent's switching cost c induces the principal to make a greater sacrifice in terms of informational payoff so as to reduce the agency cost of the designed procedure ψ . For c sufficiently large, the optimal designed procedure is completely uninformative.

Proposition 2. *Let $p(c)$ denote a selection from $\arg \max_{p > \underline{\phi}} \{I(p) - \gamma(p)\}$. Then $p(c)$ is non-decreasing in c . Furthermore, for c sufficiently large, $p(c) = 1$ and any solution of the principal's problem (P0) is such that ψ is uninformative.*

3.2 General Setting

The analysis of the general setting, with additional language constraints, is similar to that of the baseline setting; we therefore relegate it to the appendix and simply state the main result here. Before doing this, we extend some definitions previously introduced. Let

$$V_{m^\dagger}(\mu_0) := \max_{p > \phi(m^\dagger), \mu^\dagger \in \mathcal{M}(m^\dagger), \hat{\mu}} pv(\mu^\dagger) + (1-p)\hat{v}(\hat{\mu}) - [c + \gamma_{m^\dagger}(p)] \quad (\text{P}_{m^\dagger})$$

$$\text{s.t. } p\mu^\dagger + (1-p)\hat{\mu} = \mu_0,$$

with $\gamma_{m^\dagger}(p) := \frac{c\phi(m^\dagger)}{p - \phi(m^\dagger)}$. To any binary splitting $(p, \mu^\dagger, \hat{\mu})$ of μ_0 such that $\mu^\dagger \in \mathcal{M}(m^\dagger)$ and any $\alpha \in T_v(\hat{\mu})$ such that $\mu^\dagger \notin \text{supp}(\alpha)$, we can associate a splitting $\tau \in T(\mu_0)$ given by $\tau(\mu^\dagger) = p$ and $\tau(\mu) = (1-p)\alpha(\mu)$ for all $\mu \in \text{supp}(\alpha)$. Then, given an arbitrary collection $\{m_\mu\}_{\mu \in \text{supp}(\tau)}$ of distinct messages from M satisfying $m_{\mu^\dagger} = m^\dagger$ and $\mu \in \mathcal{M}(m_\mu)$ for all $\mu \in \text{supp}(\tau)$, we say

that the pair (ψ, t) is SMP-associated with $(p, \mu^\dagger, \hat{\mu})$ if (1) holds and

$$t(m_\mu) = \begin{cases} \frac{c}{p - \phi(m^\dagger)} & \text{if } \mu = \mu^\dagger \\ 0 & \text{otherwise.} \end{cases}$$

Theorem 2. *Suppose that for all $m \in M$, v is weakly concave on $\mathcal{M}(m)$, and pick $m^\dagger \in \arg \max_m V_m(\mu_0)$.⁹ There exist a binary splitting $(p, \mu^\dagger, \hat{\mu})$ of μ_0 solving (\mathbf{P}_{m^\dagger}) , and a pair (ψ, t) that is SMP-associated with this splitting. Any such (ψ, t) solves $(\mathbf{P0})$ with additional language constraints.*

4 Example

In this section, we illustrate our results in a version of the lead example in [Kamenica and Gentzkow \(2011\)](#).

The receiver is the ministry of transport, that needs to decide whether or not to build a public transportation infrastructure. The possible states of the world are ω_b and ω_n . Abusing notation slightly in this example, let μ denote the probability attached to ω_b . Assume that as long as $\mu \geq 1/2$, the ministry chooses to build. The principal on the other hand prefers building irrespective of the state of the world. For instance, the principal could be a municipality with a vested interest in building the infrastructure. We assume that the principal's payoffs are given by

$$v(\mu) = \begin{cases} 0 & \text{if } \mu \in [0, \frac{1}{2}); \\ \frac{1}{2}(1 - \eta) + \mu\eta & \text{if } \mu \in [\frac{1}{2}, 1]. \end{cases}$$

with $\eta \in (0, \frac{1}{2})$. A consultant (the agent) produces recommendations in $M = \{\text{build}, \text{do not build}\}$. Finally, $\mu_0 \in (0, \frac{1}{4})$ and we suppose that the default procedure recommends *build* and *do not build* with equal probabilities. Hence, under the default procedure the ministry never builds.

We examine the solution of the principal's problem as c increases from 0 to infinity, and illustrate our results in [Figure 2](#). At very small c , the principal behaves as in the absence of agency, and commissions a study splitting μ_0 on 0 and 1/2, inducing the recommendation *build* with probability $2\mu_0$ and the recommendation *do not build* with probability $1 - 2\mu_0$.

⁹This condition is for instance automatically satisfied whenever the messages are action recommendations and the principal is seeking to generate information so as to improve her own decision making. See [Bizzotto, Perez-Richet and Vigier \(2018\)](#).

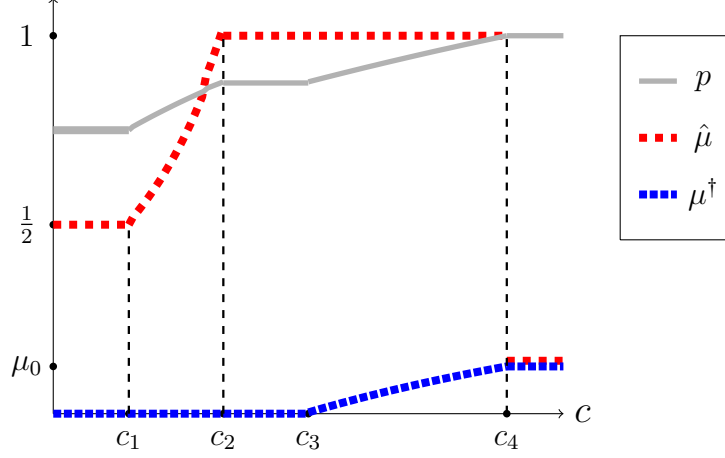


FIGURE 2: EXAMPLE

Moreover, since $1 - 2\mu_0 > 2\mu_0$, the principal rewards the agent with a payment $\frac{2c}{1-4\mu_0}$ for recommending *do not build*.

As c crosses c_1 , the principal gives up building with maximum probability in order to reduce the agency cost. Specifically, as building is more valuable to the principal in state ω_b than in state ω_n , the principal now commissions a study with a slightly lower probability of recommending *build* in state ω_n .

At $c = c_2$, the principal designs a procedure that fully reveals the state of the world.¹⁰ At this point, to further reduce the agency cost the principal must give up building in state ω_b , adding η in terms of opportunity cost. So the principal waits until $c = c_3$ in order to justify reducing the building probability any further. At $c = c_4$ the optimal procedure is uninformative: *do not build* is recommended with probability 1 irrespective of the state.

Figure 2 illustrates the optimal binary splitting of μ_0 into μ^\dagger and $\hat{\mu}$. The optimal payment probability p (whose graph we indicate by the solid curve) is obtained by maximizing $I(p) - \frac{c}{2p-1}$, where¹¹

$$I(p) = \begin{cases} \mu_0 & \text{if } p \in (1/2, 1 - 2\mu_0]; \\ \eta\mu_0 + \frac{1}{2}(1-p)(1-\eta) & \text{if } p \in [1 - 2\mu_0, 1 - \mu_0]; \\ \frac{1}{2}(1+\eta)(1-p) & \text{if } p \in [1 - \mu_0, 1]. \end{cases}$$

Lastly, we can show that if the switching cost is above c_3 , the principal then prefers to stick

¹⁰Interestingly, this shows how agency can benefit the receiver.

¹¹For all p , the principal obtains building with probability $1 - p$. Moreover, if $p \in [1 - 2\mu_0, 1 - \mu_0]$ the principal obtains building with probability 1 conditional on ω_b .

with the default procedure and save the agency cost rather than implement the optimal ψ .

5 Proof of Theorem 1

Standard information design problems allow one to conveniently restrict attention to procedures such that different messages induce different beliefs. In the kind of environment we analyze, one must first check that “merging” two messages can be done without simultaneously increasing the expected cost for the principal of making the designed procedure incentive compatible for the agent.

Lemma 1. *If (P0) admits a solution, then it also admits a solution (ψ, t) such that $\mu(m; \psi) \neq \mu(m'; \psi)$ for every $m \neq m'$.*

Proof: Let (ψ, t) solve (P0). Suppose that there exist messages $m_1 \neq m_2$ such that $\mu(m_1; \psi) = \mu(m_2; \psi)$. Pick labels such that $\phi(m_1) \leq \phi(m_2)$. Then let $\tilde{\psi}$ be the procedure defined by $\tilde{\psi}(m|\omega) = \psi(m|\omega)$ whenever $m \notin \{m_1, m_2\}$, $\tilde{\psi}(m_1|\omega) = \psi(m_1|\omega) + \psi(m_2|\omega)$ and $\tilde{\psi}(m_2|\omega) = 0$. Then $\mu(m_1; \tilde{\psi}) = \mu(m_1; \psi) = \mu(m_2; \psi)$. We also choose \tilde{t} such that $\tilde{t}(m) = t(m)$ for every $m \notin \{m_1, m_2\}$, $\tilde{t}(m_2) = 0$, while $\tilde{t}(m_1)\tilde{\psi}(m_1) = t(m_1)\psi(m_1) + t(m_2)\psi(m_2)$. By construction, (ψ, t) and $(\tilde{\psi}, \tilde{t})$ deliver the same expected payment to the agent and the same expected payoff to the principal. Hence, to show the lemma it is sufficient to show that $(\tilde{\psi}, \tilde{t})$ satisfies (IC0). This is easy to check. ■

To any pair (ψ, t) such that $\mu(m; \psi) \neq \mu(m'; \psi)$ for every $m \neq m'$ is associated a triple $(\tau_\psi, \sigma_\psi, t)$ comprising:

- (A) the splitting $\tau_\psi \in T(\mu_0)$ given by $\tau_\psi(\mu(m; \psi)) = \sum_\omega \mu_0(\omega)\psi(m|\omega)$;
- (B) the injective matching function $\sigma_\psi : \text{supp}(\tau_\psi) \rightarrow M$ given by $\sigma_\psi(\mu(m; \psi)) = m$.

Conversely, any pair (τ, σ) made up of a splitting $\tau \in T(\mu_0)$ and an injective matching function $\sigma : \text{supp}(\tau) \rightarrow M$ satisfies $(\tau, \sigma) = (\tau_\psi, \sigma_\psi)$ for some procedure ψ such that $\mu(m; \psi) \neq \mu(m'; \psi)$ for every $m \neq m'$.

By Lemma 1, we may now reformulate the problem of the principal in terms of the choice of a triple (τ, σ, t) .

Lemma 2. Consider (ψ, t) such that $\mu(m; \psi) \neq \mu(m'; \psi)$ for every $m \neq m'$. Then (ψ, t) solves (P0) if and only if $(\tau_\psi, \sigma_\psi, t)$ solves

$$\max_{\tau, \sigma, t} \sum_{\mu \in \text{supp}(\tau)} \tau(\mu) \{v(\mu) - t(\sigma(\mu))\} \quad (\text{P1})$$

$$\text{s.t.} \quad \sum_{\mu \in \text{supp}(\tau)} t(\sigma(\mu)) \{\tau(\mu) - \phi(\sigma(\mu))\} \geq c. \quad (\text{IC1})$$

Next, given an arbitrary $\tau \in T(\mu_0)$, we examine the problem of minimizing the expected payment the principal needs to make so as to implement this splitting, that is, we solve

$$\min_{\sigma, t} \sum_{\mu \in \text{supp}(\tau)} \tau(\mu) t(\sigma(\mu)) \quad (\text{CM}_\tau)$$

$$\text{s.t.} \quad \sum_{\mu \in \text{supp}(\tau)} t(\sigma(\mu)) \{\tau(\mu) - \phi(\sigma(\mu))\} \geq c. \quad (\text{IC}_\tau)$$

Lemma 3. Let $\tau \in T(\mu_0)$, $\mu^\dagger \in \arg \max_\mu \tau(\mu)$, and $m^\dagger \in \arg \min_m \phi(m)$. Then any pair (σ, t) such that $\sigma(\mu^\dagger) = m^\dagger$ and

$$t(m) = \begin{cases} \frac{c}{\bar{\tau} - \phi} & \text{if } m = m^\dagger, \\ 0 & \text{otherwise} \end{cases}$$

solves (CM_τ) . In particular, this program's value function can be written as $c + \Gamma(\tau)$, with $\Gamma(\tau) = \frac{c\phi}{\bar{\tau} - \phi}$.

Proof: We proceed in two steps. The first step fixes the matching function σ , and minimizes the cost of implementing τ given this σ . The second step optimizes over σ . Consider

$$\Gamma_\sigma(\tau) = \min_t \sum_{\mu \in \text{supp}(\tau)} \tau(\mu) t(\sigma(\mu))$$

$$\text{s.t.} \quad \sum_{\mu \in \text{supp}(\tau)} t(\sigma(\mu)) \{\tau(\mu) - \phi(\sigma(\mu))\} \geq c.$$

We can recast this program as

$$\Gamma_\sigma(\tau) = \min_{z: \text{supp}(\tau) \rightarrow \mathbb{R}_+} \sum_{\mu \in \text{supp}(\tau)} z(\mu)$$

$$\text{s.t.} \quad \sum_{\mu \in \text{supp}(\tau)} \left(\frac{\tau(\mu) - \phi(\sigma(\mu))}{\tau(\mu)} \right) z(\mu) \geq c.$$

Any solution of the latter program satisfies $z(\mu) = 0$ for all $\mu \notin \arg \max \left\{ \frac{\tau(\mu) - \phi(\sigma(\mu))}{\tau(\mu)} \right\}$, i.e. for all $\mu \notin \arg \min \frac{\phi(\sigma(\mu))}{\tau(\mu)}$. Moreover, defining $\ell_{\tau, \sigma} := \min_{\mu \in \text{supp}(\tau)} \frac{\phi(\sigma(\mu))}{\tau(\mu)}$, either $\ell_{\tau, \sigma} = 1$ in which case $\Gamma_\sigma(\tau)$ is infinite, or $\ell_{\tau, \sigma} < 1$ in which case

$$\Gamma_\sigma(\tau) = \frac{c}{1 - \ell_{\tau, \sigma}}.$$

Minimizing $\Gamma_\sigma(\tau)$ over σ therefore amounts to minimizing $\ell_{\tau, \sigma}$ over σ . ■

Lemma 3 enables us to simplify (P1) as stated in the following lemma (the proof is straightforward, and therefore omitted).

Lemma 4. *Suppose τ solves*

$$\max_{\tau} \sum_{\mu} \tau(\mu)v(\mu) - [c + \Gamma(\tau)] \quad (\text{P2})$$

and (σ, t) solves the cost minimization problem (CM_τ). Then the triple (τ, σ, t) solves (P1).

The next lemma links (P2) to the program (P) introduced in Section 3.1.

Lemma 5. *Suppose $(p, \mu^\dagger, \hat{\mu})$ solves (P) and that there exists $\alpha \in T_v(\hat{\mu})$ with $\mu^\dagger \notin \text{supp}(\alpha)$. Let $\tau(\mu^\dagger) = p$ and $\tau(\mu) = (1 - p)\alpha(\mu)$ for all $\mu \in \text{supp}(\alpha)$. Then: (i) $\tau \in T(\mu_0)$, (ii) $\bar{\tau} = p$, and (iii) τ solves (P2).*

Proof: Part (i) is trivial. We prove part (iii) below; the proof of part (ii) is similar, and relegated to the appendix. Let τ be as defined in the statement of the lemma, and suppose by way of contradiction that $\tau' \in T(\mu_0)$ does better than τ for (P2). Let $p' = \bar{\tau}'$, $\mu_a \in \arg \max_{\mu} \tau'(\mu)$, and $\mu_b = \frac{\mu_0 - p'\mu_a}{1 - p'}$. Then:

$$\begin{aligned} p'v(\mu_a) + (1 - p')\hat{v}(\mu_b) - \gamma(p') &\geq p'v(\mu_a) + (1 - p') \sum_{\mu \neq \mu_a} \frac{\tau'(\mu)v(\mu)}{1 - p'} - \gamma(p') \\ &= \sum_{\mu} \tau'(\mu)v(\mu) - \Gamma(\tau') > \sum_{\mu} \tau(\mu)v(\mu) - \Gamma(\tau) \\ &= pv(\mu^\dagger) + (1 - p) \sum_{\mu \neq \mu^\dagger} \alpha(\mu)v(\mu) - \gamma(p) \\ &= pv(\mu^\dagger) + (1 - p)\hat{v}(\hat{\mu}) - \gamma(p). \end{aligned}$$

This contradicts the optimality of $(p, \mu^\dagger, \hat{\mu})$ for program (P). ■

We are now ready to prove Theorem 1.

Proof of Theorem 1: Let $(p, \mu^\dagger, \hat{\mu})$ solve (P), and (ψ, t) be SMP-associated with this splitting. Then, by construction:

- (i) the splitting $\tau_\psi \in T(\mu_0)$ satisfies $\tau_\psi(\mu^\dagger) = p$ and $\tau_\psi(\mu) = (1-p)\alpha(\mu)$ for all $\mu \in \text{supp}(\alpha)$, where $\alpha \in T_v(\hat{\mu})$ satisfies $\mu^\dagger \notin \text{supp}(\alpha)$;
- (ii) the matching function $\sigma_\psi : \text{supp}(\tau_\psi) \rightarrow M$ satisfies $\phi(\sigma(\mu^\dagger)) = \underline{\phi}$.

We conclude from Lemma 5 that τ_ψ solves (P2) and that $\mu^\dagger \in \arg \max_\mu \tau_\psi(\mu)$, and from Lemma 3 that (σ_ψ, t) solves the corresponding cost minimization problem. By Lemma 4, the triple $(\tau_\psi, \sigma_\psi, t)$ therefore solves (P1). It ensues from Lemma 2 that (ψ, t) solves (P0).

It now only remains to show that a binary splitting $(p, \mu^\dagger, \hat{\mu})$ of μ_0 solving (P) exists, and that so does a pair (ψ, t) that is SMP-associated with it. We relegate these steps of the proof to the appendix. ■

6 Conclusion

We proposed a tractable model of information design in which the task of acquiring information is delegated to an agent who must be incentivized in order to follow the information acquisition procedure designed by the principal. While our model assumes that the contractibles are the messages that an information structure generates, this framework can in fact capture other natural assumptions about contractibility. We provided a general methodology to tackle such problems, and examined how moral hazard transforms the design problem faced by the principal.

Appendix

Omitted Step of the Proof of Lemma 5: Let τ be as defined in the statement of the lemma. Here, we prove that that $\bar{\tau} = p$. Suppose, by way of contradiction, that $\bar{\tau} > p$. Define $p' := \bar{\tau}$. Let $\tau(\mu_a) = p'$ and $\mu_b := \frac{\mu_0 - p'\mu_a}{1-p'}$. Then:

$$\begin{aligned}
p'v(\mu_a) + (1-p')\hat{v}(\mu_b) - [c + \gamma(p')] &> p'v(\mu_a) + (1-p') \sum_{\mu \neq \mu_a} \frac{\tau(\mu)v(\mu)}{1-p'} - [c + \gamma(p)] \\
&= \sum_{\mu} \tau(\mu)v(\mu) - [c + \gamma(p)] \\
&= pv(\mu^\dagger) + (1-p) \sum_{\mu \neq \mu^\dagger} \alpha(\mu)v(\mu) - [c + \gamma(p)] \\
&= pv(\mu^\dagger) + (1-p)\hat{v}(\hat{\mu}) - [c + \gamma(p)].
\end{aligned}$$

This contradicts the optimality of $(p, \mu^\dagger, \hat{\mu})$. ■

Lemma 6. *If $(p, \mu^\dagger, \hat{\mu})$ solves (P) and $p < 1$ then, for all $\alpha \in T_v(\hat{\mu})$, we have $\alpha(\mu^\dagger) = 0$.*

Proof: Let $(p, \mu^\dagger, \hat{\mu})$ solve (P), with $p < 1$, and $\alpha \in T_v(\hat{\mu})$. Suppose by way of contradiction that $\alpha(\mu^\dagger) > 0$. Define $p' := p + (1-p)\alpha(\mu^\dagger)$, and $\tilde{\mu} := \frac{\mu_0 - p'\mu^\dagger}{1-p'}$. Since $\mu_0 = p\mu^\dagger + (1-p)\hat{\mu}$ and $\sum_{\mu} \alpha(\mu)\mu = \hat{\mu}$, note that

$$\tilde{\mu} = \frac{(1-p) \sum_{\mu \neq \mu^\dagger} \alpha(\mu)\mu}{1-p'}. \quad (2)$$

Then, using (2) and $1-p' = (1-p)(1-\alpha(\mu^\dagger))$:

$$\begin{aligned}
p'v(\mu^\dagger) + (1-p')\hat{v}(\tilde{\mu}) &= pv(\mu^\dagger) + (1-p)\alpha(\mu^\dagger)v(\mu^\dagger) + (1-p)(1-\alpha(\mu^\dagger))\hat{v}(\tilde{\mu}) \\
&\geq pv(\mu^\dagger) + (1-p)\alpha(\mu^\dagger)v(\mu^\dagger) + (1-p)(1-\alpha(\mu^\dagger)) \sum_{\mu \neq \mu^\dagger} \frac{(1-p)\alpha(\mu)v(\mu)}{1-p'} \\
&= pv(\mu^\dagger) + (1-p)\alpha(\mu^\dagger)v(\mu^\dagger) + \sum_{\mu \neq \mu^\dagger} (1-p)\alpha(\mu)v(\mu) \\
&= pv(\mu^\dagger) + (1-p) \sum_{\mu} \alpha(\mu)v(\mu) = pv(\mu^\dagger) + (1-p)\hat{v}(\hat{\mu}).
\end{aligned}$$

This contradicts the optimality of $(p, \mu^\dagger, \hat{\mu})$ for program (P). ■

Omitted Steps of the Proof of Theorem 1: First, we show that there exists a solution to (P). By choosing $p = 1$ in (P), we achieve the value $v(\mu_0) - c - \gamma(1)$. Furthermore $\hat{v}(\mu_0)$ is

an upper bound for the informational payoff $pv(\mu^\dagger) + (1-p)\hat{v}(\hat{\mu})$. For p sufficiently close to $\underline{\phi}$, the agency cost is so high that the principal would not want to choose p even if she could attain her best informational payoff by doing so. This is the case if

$$\hat{v}(\mu_0) - \gamma(p) < v(\mu_0) - \gamma(1),$$

or, equivalently, if

$$p < \underline{p} := \underline{\phi} + \frac{c\underline{\phi}}{\hat{v}(\mu_0) - v(\mu_0) + \frac{c\underline{\phi}}{1-\underline{\phi}}}.$$

Hence, if $\underline{p} < 1$, we can rewrite (P) as a maximization problem over the set of triples $(p, \mu^\dagger, \hat{\mu}) \in [\underline{p}, 1] \times \Delta\Omega^2$ that satisfy (BP). This set is compact, and the objective function in (P) is upper semicontinuous in $(p, \mu^\dagger, \hat{\mu})$. We conclude that if $\underline{p} < 1$ then a solution to (P) exists (see, for example, [Aliprantis and Border, 2006](#), theorem 2.43). The only remaining case is if $\underline{p} \geq 1$. In this case, the principal can not do better than choosing an uninformative procedure, and a solution to (P) exists with $p = 1$.

Next, we show that given $(p, \mu^\dagger, \hat{\mu})$ solving (P) we can find a pair (ψ, t) that is SMP-associated with it. This is immediate if $p = 1$. If $p < 1$, all we need to do is to show the existence of $\alpha \in T_v(\hat{\mu})$ with $\mu^\dagger \notin \text{supp}(\alpha)$. However, by [Lemma 6](#), in this case any $\alpha \in T_v(\hat{\mu})$ satisfies $\alpha(\mu^\dagger) = 0$. ■

Proof of Proposition 1: Consider a solution (ψ, t) of (P0) that is SMP-associated with the binary splitting $(p, \mu^\dagger, \hat{\mu})$ of μ_0 solving (P). The splitting $\tau_\psi \in T(\mu_0)$ induced by the procedure ψ satisfies $\tau_\psi(\mu^\dagger) = p$, and $\tau_\psi(\mu) = (1-p)\alpha(\mu)$ for all $\mu \neq \mu^\dagger$, where $\alpha \in T_v(\hat{\mu})$. Note that $\bar{\tau}_\psi \geq p$.

Suppose by way of contradiction that we can find $\tilde{p} \in (\underline{\phi}, 1]$ with $I(\tilde{p}) - \gamma(\tilde{p}) > I(p) - \gamma(p)$. Let $\tau' \in T(\mu_0)$ achieve the value $I(\tilde{p})$. Then, by definition of the corresponding program,

$p' := \bar{\tau}' \geq \tilde{p}$. Next, let $\mu_a \in \arg \max_{\mu} \tau'(\mu)$, and $\mu_b := \frac{\mu_0 - p' \mu_a}{1 - p'}$. We then have

$$\begin{aligned}
p'v(\mu_a) + (1 - p')\hat{v}(\mu_b) - \gamma(p') &\geq p'v(\mu_a) + (1 - p') \sum_{\mu \neq \mu_a} \frac{\tau'(\mu)v(\mu)}{1 - p'} - \gamma(p') \\
&= \sum_{\mu} \tau'(\mu)v(\mu) - \gamma(p') = I(\tilde{p}) - \gamma(p') \\
&\geq I(\tilde{p}) - \gamma(\tilde{p}) > I(p) - \gamma(p) \geq \sum_{\mu} \tau_{\psi}(\mu)v(\mu) - \gamma(p) \\
&= pv(\mu^{\dagger}) + (1 - p) \sum_{\mu \neq \mu^{\dagger}} \alpha(\mu)v(\mu) - \gamma(p) \\
&= pv(\mu^{\dagger}) + (1 - p)\hat{v}(\hat{\mu}) - \gamma(p).
\end{aligned}$$

This contradicts the optimality of $(p, \mu^{\dagger}, \hat{\mu})$ for program (P). ■

Proof of Proposition 2: The first part of the proposition is a direct consequence of the Monotone Selection Theorem of [Milgrom and Shannon \(1994\)](#). For the second part, if $\hat{v}(\mu_0) = v(\mu_0)$ then the result is immediate, so suppose $\hat{v}(\mu_0) > v(\mu_0)$. Then, by definition of \underline{p} in the proof of Theorem 1, we have $\underline{p} \geq 1$ whenever

$$c \geq \bar{c} = \frac{(1 - \underline{\phi})(\hat{v}(\mu_0) - v(\mu_0))}{\underline{\phi}^2},$$

and the solution of (P) must then be such that $p = 1$. ■

Proof of Theorem 2: Let $m^{\dagger} \in \arg \max_m V_m(\mu_0)$. The existence of a binary splitting $(p, \mu^{\dagger}, \hat{\mu})$ of μ_0 solving (P _{m^{\dagger}}) follows from arguments similar to those developed in the proof of Theorem 1. We now show that given $(p, \mu^{\dagger}, \hat{\mu})$ solving (P _{m^{\dagger}}) we can find a pair (ψ, t) that is SMP-associated with it. This is immediate if $p = 1$. If $p < 1$, all we need to do is to show the existence of $\alpha \in T_v(\hat{\mu})$ with $\mu^{\dagger} \notin \text{supp}(\alpha)$ and such that, for all $m \in M$, $\text{supp}(\alpha) \cap \mathcal{M}(m)$ contains at most one element. Arguments identical to those in the proof of Lemma 6 show that any $\alpha \in T_v(\hat{\mu})$ satisfies $\mu^{\dagger} \notin \text{supp}(\alpha)$. Moreover, since v is weakly concave on $\mathcal{M}(m)$, choosing α such that $\text{supp}(\alpha) \cap \mathcal{M}(m)$ contains at most one element for all $m \in M$ is without loss of generality.

Next, pick $(p, \mu^{\dagger}, \hat{\mu})$ solving (P _{m^{\dagger}}) and a pair (ψ, t) that is SMP-associated with it. Consider an arbitrary pair (ψ', t') satisfying (IC0) as well as the language constraints. Using arguments similar to those in Lemma 3, we can without loss of generality assume that t' rewards the

agent at a single message, that we denote m_a . Then, let $\mu_a := \mu(m_a; \psi')$, $p_a := \sum_{\omega} \psi'(m_a|\omega)$, and $\mu_b := \frac{\mu_0 - p_a \mu_a}{1 - p_a}$. As $(p, \mu^\dagger, \hat{\mu})$ solves (\mathbf{P}_{m^\dagger}) ,

$$pv(\mu^\dagger) + (1 - p)\hat{v}(\hat{\mu}) - [c + \gamma_{m^\dagger}(p)] \geq p_a v(\mu_a) + (1 - p_a)\hat{v}(\mu_b) - [c + \gamma_{m_a}(p_a)].$$

The left-hand side of this inequality is the expected payoff of the principal from choosing (ψ, t) . The expected payoff of the principal from choosing (ψ', t') is bounded from above by right-hand side of this inequality. ■

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